Kindergarten Mathematics, Quarter 4, Unit 4.1

Developing Number Sense Through Counting and Comparison

Overview

Number of instructional days: 5 (1 day = 45–60 minutes)

Content to be learned

- Count up to 20 objects arranged in a line to answer "how many?"
- Count up to 20 objects in a rectangular or circular array to answer "how many?"
- Count up to 10 objects in a scattered configuration to answer "how many?"
- Count out the correct number of objects when given a number from 1-20.
- Compare two numbers between 1 and 10 presented as written numerals without a manipulative or pictorial model.

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Mathematically proficient students justify their conclusions, communicate them to others, and respond to the arguments of others.
- They reason inductively, making plausible arguments that take into account the context.
- Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades.

Attend to precision.

- Mathematically proficient students calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.
- In the elementary grades, students give carefully formulated explanations to each other.

Look for and make use of structure.

- Mathematically proficient students look closely to discern a pattern or structure.

Essential questions

- How can you show which set of objects matches this number (0–20) using manipulatives, drawings, models and number lines?
- How can you be sure you are counting correctly?
- How can you count these objects correctly? (objects scattered or arranged in an array)

- How can you count these objects correctly? (objects arranged in a line)
- How do you know which number is higher/lower (or more/less)?

Written Curriculum

Common Core State Standards for Mathematical Content

Counting and Cardinality	K.CC

Count to tell the number of objects.

K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Compare numbers.

K.CC.7 Compare two numbers between 1 and 10 presented as written numerals.

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

According to the Rhode Island Early Learning Standards (www.ride.ri.gov/els/math.asp), students entering kindergarten may have had experience with counting and number vocabulary as part of play. They may use one-to-one correspondence in counting objects and matching groups of objects. They may begin to associate a number of objects with the names and symbols for numbers.

Current Learning

Kindergarten students count as many as 20 objects arranged in a line or in a rectangular or circular array. They count as many as 10 objects in a scattered configuration. When given a number from 1-20, they are able to make a matching set of that many objects. They compare two written numerals from 1-10 and are able to tell which one means more or less. The level of instruction in this unit is provided at the reinforcement level.

The routines established for counting by ones, counting with one-to-one correspondence, and comparing groups of objects should continue in this unit.

Future Learning

In grade 1, students will use counting strategies to solve word problems involving comparing, addition, and subtraction. They will use objects, drawings, and equations with a symbol for an unknown number to represent the problem.

Additional Findings

According to the *Progressions for the Common Core State Standards in Mathematics* (K, Counting & Cardinality, May 2011), counting objects arranged in a line is easiest; with more practice, students learn to count objects in more difficult arrangements, such as rectangular arrays (they need to insure they reach every row or column and do not repeat rows or columns); circles (they need to stop just before the object they started with); and scattered configurations (they need to make a single path through all of the objects). (p.4)

Also, the standards about comparing numbers (K.CC.6 & K.CC.7) focus on students identifying which of two groups has more than (or fewer than, or the same amount as) the other. Students first learn to match the objects in two groups to see if there are any extra and then to count the objects in each group and use their knowledge of the count sequence to decide which number is greater than the other (the number farther along in the count sequence). Students learn that even if one group looks as if it has more objects (e.g. has some extra sticking out), matching or counting may reveal a different result. Comparing numbers progresses in Grade 1 to adding and subtracting in comparing situations (finding out "how many more" or "how many less" and not just "which is more" or "which is less").

Benchmarks For Science Literacy states that counting and counting games in which students are challenged to count forward and backward, skip count, match numbers and things, guess how many things are in a set, and count to see who is right are popular with students and help them become comfortable with numbers. These counting games should be extended to include having students compare/combine/equalize and change numbers as well as 'take away' and 'add to'" (p. 211).

They also state that "concrete objects should be employed routinely to help children discover and explain symbolic relationships. Students should come to see that numbers and shapes can be used to describe many things in the world around them. Eventually, they should come to realize that just as letters and words make up language in reading and writing, number and shapes make up a language in mathematics." (p. 36).

Kindergarten Mathematics, Quarter 4, Unit 4.2

Addition and Subtraction to 10

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Show addition and subtraction using real-world situations and incorporate equations.
- Solve addition and subtraction word problems within 10 with representation.
- Decompose numbers within 10 using manipulatives, drawings, and equations.
- Determine a number that can be added a given number to make 10 using manipulatives, drawings, and equations.
- Add and subtract within 5 with automaticity.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Mathematically proficient students start by explaining to themselves the meaning of a problem
- Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem.

Look for and express regularity in repeated reasoning.

- With support, students continually evaluate the reasonableness of their intermediate results.

Essential questions

- How do you know if you need to add or subtract to solve this problem?
- How can you use counters and a ten-frame to break (a given number up to 10) into two smaller groups?
- How can you write an equation or use drawings or manipulatives to show this problem?
- How can you use objects or a drawing to find the missing number in this equation?
- How can you add or subtract these two numbers (up to 5)?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking	K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings[2], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

[2] Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

K.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

K.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5 = 2 + 3$ and $5 = 4 + 1$).

K.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.

K.OA.5 Fluently add and subtract within 5.

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

According to the Rhode Island Early Learning Standards, students entering kindergarten may "begin to use numbers and counting as a means for solving problems and they may use words such as more than, less than and add/subtract to express some number concepts."

According to *Progressions for the Common Core State Standards in Mathematics* (K, Counting & Cardinality, May 2011), "students may bring from home different ways to show numbers with their fingers and to raise (or lower) them when counting." (p. 8)

Current Learning

Students in kindergarten use real-world situations, objects, pictorial representations, and equations to solve addition and subtraction problems up to 10, and learn to fluently add and subtract within 5. They decompose numbers within 10 using representations and equations. Students determine a number that is added to another number to equal 10. This unit is taught at the reinforcement level with the inclusion of equations, and should incorporate drill and practice activities such as games and real-world problems. Routines of counting and using objects and drawings to solve word problems should continue in this unit.

Future Learning

In grade 1, student will solve addition and subtraction problems using up to three whole numbers within 20, and will add and subtract fluently within 10. The will apply the commutative and associative properties of addition and determine the unknown number in an addition and subtraction problems.

Additional Findings

Principles and Standards for School Mathematics states that "an understanding of addition and subtraction can be generated when young students solve 'joining' and 'take away' problems by directly modeling the situation or by using counting strategies, such as counting on or counting back. Students develop further understandings of addition when they solve missing addend problems that arise from stories or real situations" (p. 83).

According to *Progressions for the Common Core State Standards in Mathematics* (K, Counting & Cardinality, May 2011), "students act out adding and subtracting situations by representing quantities in the situation with objects, their fingers, and math drawings. To do this, students must mathematize a real-world situation, focusing on the quantities and their relationships rather than non-mathematical aspects of the situation. Situations can be acted out and/or presented with pictures or words. Math drawings facilitate reflection and discussion because they remain after the problem is solved. These concrete methods that show all of the objects are called Level 1 methods." (p. 8)

It also states that "students learn and use mathematical and non-mathematical language, especially when they make up problems and explain their representation and solution." (p. 8)

Also, "Put Together/Take Apart situations with both addends unknown play an important role in kindergarten because they allow students to explore various compositions that make each number." (p.10)

"Equations with one number on the left and an operation on the right (e.g. 5 = 2+3 to record a group of 5 things decomposed as a group of 2 things and a group of 3 things) allow students to understand equations as showing in various ways that the quantities on both sides have the same value." (p. 10)
"Experience with decompositions of numbers and with Add To and Take From situations enables students to begin to fluently add and subtract within 5." (p. 11)

Kindergarten Mathematics, Quarter 4, Unit 4.3

Foundations of Base Ten

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Compose numbers from 11-19 into tens and ones using manipulatives or pictorial representations.
- Decompose numbers from 11-19 into tens and ones using manipulatives or pictorial representations.
- Understand that as one counts forward from 11-19, the manipulative group or drawing that represents the ones increases by one object.
- Show a drawing or equation representing the groups of tens and ones.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Mathematically proficient students make sense of quantities and their relationships in problem situations.
- Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Look for and make use of structure.

- Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that 14=10+4 (1 tens and 4 ones) and 15=10+5 (1 ten and 5 ones).

Essential questions

- How can you use manipulatives or pictures to create a set of 15 (or other number from 11–19) objects showing the tens and ones?
- How can you use manipulatives or pictures to take apart a set of 15 (or other number from 11–19) objects to show the tens and ones?
- What happens to the groups of tens and ones as you count up from 11–19?
- How can you use a drawing or equation to show your groups of tens and ones?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten	K.NBT

Work with numbers 11–19 to gain foundations for place value.

K.NBT.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

Children may enter kindergarten with the ability to count through the teen numbers accurately, and be able to count out a given number of teen objects.

Current Learning

Students in kindergarten learn to create and take apart groups of objects between 11-19, showing the correct number of tens and ones. As they count up from 11-19, they see a pattern of how the number in the ones group changes by one object. They use drawings or equations to represent the number of tens and ones for a given number. Instruction in this unit is delivered at the developmental level.

Routines of oral counting, counting objects, and composing/decomposing numbers should continue in this unit.

Future Learning

In grade 1, students explore place value in more depth. They learn that the digits of a two-digit number represents the number of tens and ones, they compare two two-digit numbers, and record using the symbols >, =, and <.

Additional Findings

According to *Progressions for the Common Core State Standards in Mathematics* (K–5, Numbers and Operations in Base Ten, May 2011), "a difficulty in the English-speaking world is that the words for teen numbers do not make their base-ten meanings evident. For example, "eleven" and "twelve" do not sound like "ten" and "one and "ten" and "two." The numbers "thirteen, fourteen, fifteen ... nineteen" reverse the order of the ones and tens digits by saying the ones digits first. Also, "teen" must be interpreted as meaning "tens" and the prefixes "thir" and "fif" do not clearly say "three" and "five." In contrast, the corresponding East Asian number words are "ten one, ten two, ten three," and so on, fitting directly with the base ten structure and drawing attention to the role of ten. (p. 5)

Kindergarten Mathematics, Quarter 4, Unit 4.4

Analyzing, Comparing, and Composing Shapes

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Use materials (i.e. sticks, clay) to create shapes in the environment
- Draw shapes using various materials (sand, shaving cream, drawing/writing tools)
- Combine shapes to form larger shapes (i.e. join two triangles with full sides touching to make a rectangle)
- Arrange shapes in various ways to create a design or picture.

Mathematical practices to be integrated

Look for and make use of structure.

- Mathematically proficient students look closely to discern a pattern or structure.
- They can see complicated things as single objects or as being composed of several objects.

Essential questions

- How can you use these materials (i.e. sticks, clay) to create shapes in the world?
- What kind of shapes do you have in your drawing and where would you see those shapes (outside, in the classroom, at home)?
- How can you use two or more shapes to build a rectangle (or other shape)?
- How can you use pattern blocks, tangrams, or geoboards to create a design or picture?

Written Curriculum

Common Core State Standards for Mathematical Content

Geometry	**K.G**

Analyze, compare, create, and compose shapes.

K.G.5 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.

K.G.6 Compose simple shapes to form larger shapes. For example, "Can you join these two triangles with full sides touching to make a rectangle?"

Common Core Standards for Mathematical Practice

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.Clarifying the Standards

Prior Learning

Students entering kindergarten are expected to recognize similarities and difference among everyday objects. According to the Rhode Island Early Learning Standards (www.ride.ri.gov/els/resources.asp), they may also "experiment with patterns and shapes" and "describe and name common shapes found in the natural environment."

Current Learning

Kindergarten students learn to recognize and describe shapes in the environment. They learn to identify shapes as either two-dimensional or three-dimensional, then build and draw shapes in the world. Students combine shapes to form larger shapes and use shapes to create designs and pictures. Instruction is at the reinforcement level for building shapes in the world from components. Instruction is at the developmental level for composing simple shapes to form larger shapes and using shape materials to create designs and pictures.

The routines of locating shapes in the environment and recognizing/naming two-dimensional and three-dimensional shapes should continue in this unit.

Future Learning

In grade 1, students will compose two-dimensional or three-dimensional shapes to create a new composite shape, then use that composite shape to form a new shape (i.e., combine a square and a triangle to make a house, then understand that a new shape has been created. Students do not need to learn formal names for the new shape, such as pentagon).

Additional Findings

Benchmarks for Science Literacy states, "by the end of second grade, students should know that circles, squares, triangles, and other shapes can be found in things in nature and in things that people build" (p. 26). The book further states that "by the end of fifth grade, students should know that mathematics is the study of many kinds of patterns, including numbers and shapes and operations on them" (p. 27).

A Research Companion to Principles and Standards of School Mathematics states, "Piaget and Inhelder's first theme that children's representation of space is constructed from active manipulation of their spatial environment has been supported. Children's ideas about shapes do not come from passive looking. Instead, they come as children's bodies, hands, eyes and minds engage in active construction. Children need to explore shapes extensively to fully understand them. Merely seeing and naming picture is not sufficient. Finally, they have to explore the parts and attributes of shapes" (p. 152).

Principles and Standards for School Mathematics states that:

Pre-K–2 geometry begins with describing and naming shapes. Young students begin by using their own vocabulary to describe objects, talking about how they are alike and how they are different. Teachers must help students gradually incorporate conventional terminology into their descriptions of two- and three-dimensional shapes. However, terminology itself should not be the focus of the pre-K–2 geometry program. The goal is that early experiences with geometry lay the foundation for more formal geometry in later grades. Using terminology to focus attention and to clarify ideas during discussions can help students build that foundation.

Teachers must provide materials and structure the environment appropriately to encourage students to explore shapes and their attributes. For example, young students can compare and sort building blocks as they put them away on shelves, identifying their similarities and differences. They can use commonly available materials such as cereal boxes to explore attributes of shapes or folded paper to investigate symmetry and congruence. Students can create shapes on geo-boards or dot paper and represent them in drawings, block constructions, and dramatizations.

Students also learn about geometric properties by combining or cutting apart shapes to form new shapes. (pp. 97–98).